

(540)

$$2x_1 + x_2 + x_3 = 0$$

$$-6x_1 + 3x_2 + 9x_3 = 6$$

$$2x_1 + 2x_2 + 3x_3 = 1$$

$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ -6 & 3 & 9 & 6 \\ 2 & 2 & 3 & 1 \end{array} \begin{array}{l} \\ (+3\text{I}) \\ (-\text{I}) \end{array}$$

$$\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 6 & 12 & 6 \\ 0 & 1 & 2 & 1 \end{array} \quad (-\frac{1}{6}\text{II})$$

$$\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 6 & 12 & 6 \\ 0 & 0 & 0 & 0 \end{array} \left. \vphantom{\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 6 & 12 & 6 \\ 0 & 0 & 0 & 0 \end{array}} \right\} \rightarrow \begin{array}{l} 2x_1 + x_2 + x_3 = 0 \\ 6x_2 + 12x_3 = 6 \\ \cancel{0 = 0} \end{array}$$

x_3 beliebig, $x_3 := t \in \mathbb{R}$

$$6x_2 + 12t = 6 \Leftrightarrow x_2 = \frac{6-12t}{6} \Leftrightarrow x_2 = 1-2t$$

$$2x_1 + \underbrace{1-2t+t}_{=1-t} = 0 \Leftrightarrow x_1 = \frac{t-1}{2}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{t-1}{2} \\ 1-2t \\ t \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} \frac{1}{2} \\ -2 \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}$$