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geg.: $P(X \in (-\infty, b))$ für alle $b \in \mathbb{R}$

ges.: $P(X \in [a, \infty))$, $P(X \in (-\infty, b])$, $P(X \in (a, \infty))$,
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 $P(X \in [a, b))$, $P(X \in (a, b])$

$$\begin{aligned} P(X \in [a, \infty)) &= 1 - P(X \notin [a, \infty)) \\ &= 1 - P(X \in (-\infty, a)) \end{aligned}$$

$$P(X \in [a, b)) = P((X \in (-\infty, b)) \cap (X \in [a, \infty)))$$

$$\begin{aligned} (P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ \Leftrightarrow P(A \cap B) &= P(A) + P(B) - P(A \cup B)) \end{aligned}$$

$$= P(X \in (-\infty, b)) + P(X \in [a, \infty)) - \underbrace{P(X \in \mathbb{R})}_{=1}$$

$$\begin{aligned} P(X \in (a, \infty)) &= P\left(X \in \left(\left(\bigcup_{k=1}^{\infty} \left[a + \frac{1}{k+1}, a + \frac{1}{k}\right)\right) \cup [a+1, \infty)\right)\right) \\ &= \sum_{k=1}^{\infty} P(X \in [a + \frac{1}{k+1}, a + \frac{1}{k})) + P(X \in [a+1, \infty)) \end{aligned}$$

$$P(X \in (-\infty, b]) = 1 - P(X \in (b, \infty))$$

$$\begin{aligned} P(X \in (a, b)) &= P((X \in (-\infty, b)) \cap (X \in (a, \infty))) \\ &= P(X \in (-\infty, b)) + P(X \in (a, \infty)) - \underbrace{P(X \in \mathbb{R})}_{=1} \end{aligned}$$